Voting for a Group Decision: How I Was One Voting-lecture Away From "Winning" the Group Decision

This autumn artist Billie Eilish announced her world tour happening summer of 2020. An eager Billie Eilish-fan in my friend group quickly wrote a message in the group chat; "Who wants to join a Billie Eilish concert next summer *heart-eye emoji* ??!!". Five of us wanted to join, and we concluded with three possible destinations we could go, that were all during a week-end: Amsterdam (Netherlands), Milan (Italy), Paris (France).

A Facebook Messenger Poll was created. In the messenger poll-system the rules are simple; you can vote once per option, and you can vote for as many or few options as you like.Most people (in my experience) will simply vote for all options they might be interested in, or willing to accept, but not vote for any option that is a definite no.

For the sake of further discussion, I have created a table of how the different people in my friend group preferred the different destinations:

Preference	**Friend A**	**Friend B**	**Friend C**	**Friend D**	**Me**
1	Paris	Paris	Amsterdam	Milan	Amsterdam
2	Milan	Milan	Paris	Amsterdam	Milan
3	Amsterdam	Amsterdam	Milan	Paris	Paris

Everyone voted for their two first options,
The poll ended like this:
Milan: 4 votes.
Paris: 3 votes.
Amsterdam: 3 votes.

Milan obviously won, though it wasn't my first choice. My first choice was Amsterdam. Now, let's look at this way of voting, and how Amsterdam could have won, If I had only knew better.

The Facebook Messenger Voting System
The messenger voting system is hard to categorize as an particular type of voting system. In some way it is similar to a positional voting system, but it also differs since the voters themself can choose a strategy for voting, that is, how many to vote for; a voter can vote anywhere for just his top pick, to all possible options that he has any interest in at all. I find it most similar to a Approval voting system. Approval Voting System is defined as "each voter selects a subset of the candidates (where the empty set means the voter abstains) and the candidate(s) with selected by the most voters wins" (Pacuit, 2019). This definition agree with the fact that a member of a messenger-chat can choose to not vote at all.

Pairwise Majority Voting:
In this voting-method voting is always happening between pairs, as the name would suggest. We use a group preference relation > as follows: For each pair of alternatives (destinations in our case) X and Y, X>Y if X is preferred over Y by a majority of the voters, or Y>X is Y is preferred over X by a majority of voters. So based on the table above, we get the following relations:

Milan vs. Paris: Paris>Milan
Milan vs. Amsterdam: Milan>Amsterdam
Paris vs. Amsterdam: Amsterdam>Paris

We end up with a weird cyclic relation, looking something like Paris>Milan>Amsterdam>Paris, which doesn't make any sense. This is called Condercet's Paradox, which is the possibility of non-transitive group-rankings arising from transitive individual preferences. It was first noted by the french philosopher and mathematician Nicolas De Condorcet in the late 18th century (Easley and Kleinberg, 2010, p. 741)

When we end up with cyclical societal preferences, no winner can be named. Therefor, it turns out that we could not have decided to which city to go to using Pairwise Majority Voting, so let's move on to the next method.

Positional Voting
With a positional voting system the options receive a weight based on their rank position from each voter, and the option with the highest overall weight wins. There are several different positional voting systems, based on how the different priorities are weighted. In Borda count the lowest position gets weight 0, next lowest 1 and least lowest gets weigh k-1 (where k is the number of alternatives)(Easley and Kleinberg, 2010, p. 745). The weights corresponds to the number of destinations ranked lower than the destination. Borda count is named after the 18th-century French mathematician Jean-Charles de Borda who first formulated it (Easley and Kleinberg, 2010, p. 745).

In our case, the weights will look like this:

Place	**Weight**	**Friend A**	**Friend B**	**Friend C**	**Friend D**	**Me**
1	**2**	Paris	Paris	Amsterda m	Milan	Amsterdam
2	**1**	Milan	Milan	Paris	Amsterdam	Milan
3	**0**	Amsterdam	Amsterdam	Milan	Paris	Paris

We then sum up the weights to each destination:

Milan: 1+1+0+2+1=5
Paris: 2+2+1+0+0=5
Amsterdam: 0+0+2+1+2=5
All the destinations end up with the same total weight, and neither can be named a winner. But there is something in my power I can do to change the outcome, which is rearranging my preferences. So called strategic misreporting of preferences is a pathology in positional voting systems (Easley and Kleinberg, 2010, p. 747). Easley and Kleinberg writes that it's a problem that "arises from the fact that competition for top spots in the group ranking can depend critically on the rankings of alternatives that are further down the list".

Lets try some other positions of mine:

Place	**Weight**	**Friend A**	**Friend B**	**Friend C**	**Friend D**	**Me**
1	**2**	Paris	Paris	Amsterda m	Milan	Amsterdam
2	**1**	Milan	Milan	Paris	Amsterdam	Paris
3	**0**	Amsterdam	Amsterdam	Milan	Paris	Milan

Milan: 1+1+0+2+0=4
Paris: 2+2+1+0+1=6
Amsterdam: 0+0+2+1+2=5

And another position:

Place	**Weight**	**Friend A**	**Friend B**	**Friend C**	**Friend D**	**Me**
1	**2**	Paris	Paris	Amsterda m	Milan	Amsterdam
2	**1**	Milan	Milan	Paris	Amsterdam	Paris
3	**0**	Amsterdam	Amsterdam	Milan	Paris	Milan

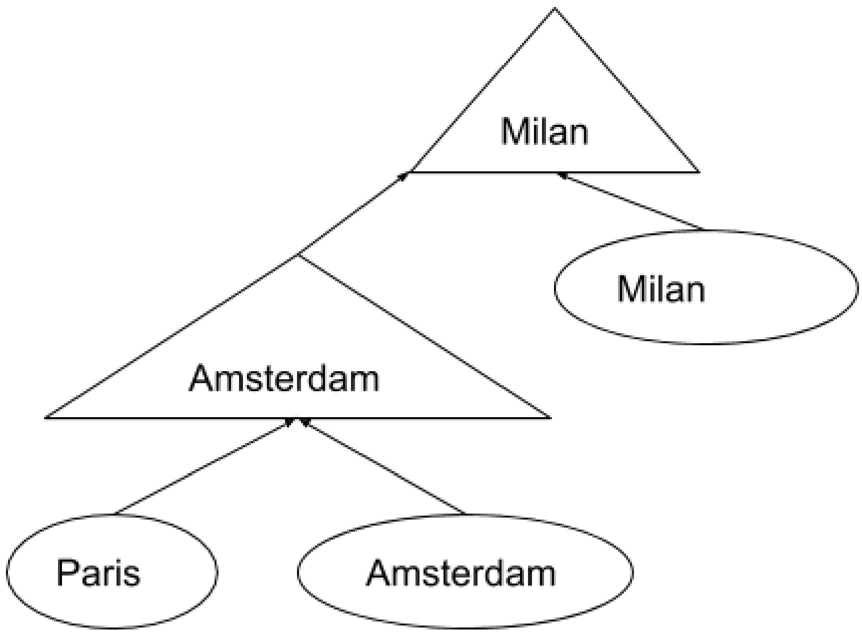
Milan: 1+1+0+2+0=4
Paris: 2+2+1+0+1=6
Amsterdam: 0+0+2+1+2=5

So none of the different positions of the destinations could make Amsterdam be the overall winner. We move on to a third voting system.

Elimination tournament:
In an elimination tournament the alternatives is arranged in some order, and then eliminate them one-by-one using the majority rule (Easley and Kleinberg, 2010, p.742). The winner of the final comparison is also the winner of the whole

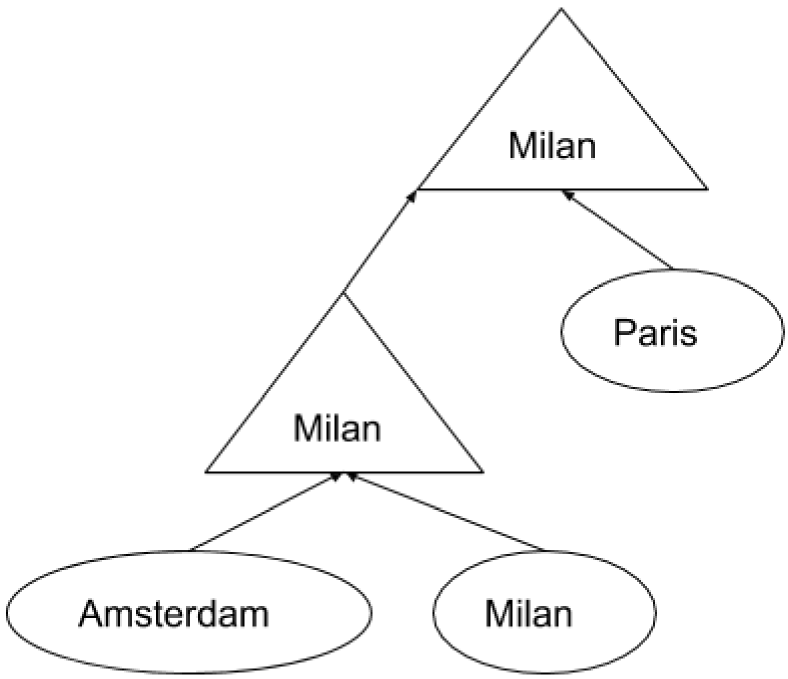
tournament. In our case, we can arrange the tournament in three different ways, lets see if any of them will make Amsterdam the winner.

Let’s start by first comparing Paris and Amsterdam, and then introducing Milan:



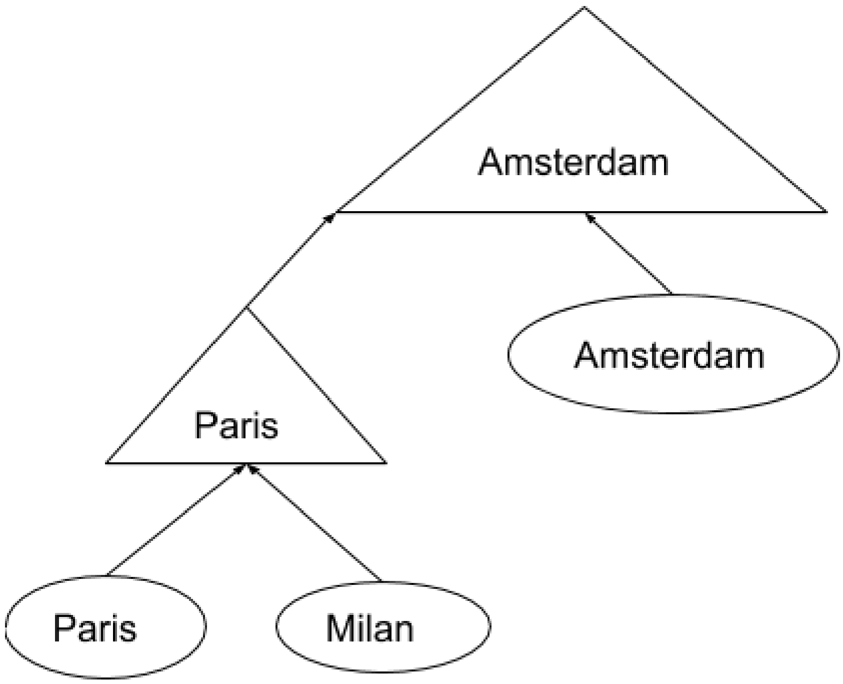
Milan turns out to be the winner, just like in our Messenger-poll.

Now we start by comparing Amsterdam and Milan, and then introduce Paris last:



Again, Milan is the winner.

Maybe third time's a charm, let's start by comparing Paris and Milan, and then introduce Amsterdam:



Yes, my top-rated destination is now on the winner of the elimination tournament!

This shows how powerful it can be to know voting theory. Though where to travel for a weekend trip is a trivial choice, the same voting mechanisms can be applied to much more critical and important elections. In recent years particularly, discussions about voting and democracy has been highly relevant. E.g. in the upcoming general election in the UK, BBC (2019) reports that "some campaign groups are suggesting people use tactical-voting websites, to help them decide which candidate to support", which caused a huge stir.
Much more could be said about voting theory and tactical voting, but until then… Bon Voyage!

Sources:

- Easley, D. and Kleinberg, J. (2010) Networks, Crowds and Markets: Reasoning about a Highly connected World. Cambridge University Press. Available from: https://www.cs.cornell.edu/home/kleinber/networks-book/networks-book-ch23.pdf (Accessed: 03.11.2019).
- Pacuit, Eric, "Voting Methods", *The Stanford Encyclopedia of Philosophy* (Fall 2019 Edition), Edward N. Zalta (ed.), Available from: https://plato.stanford.edu/archives/fall2019/entries/voting-methods/ (Accessed: 07.11.19).
- BBC. (2019) General Election 2019: What is tactical voting? Available from: https://www.bbc.com/news/uk-politics-50249649 (Accessed: 10.11.19)